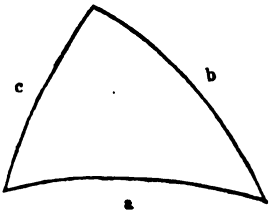
Art. 2.—Investigations of the Rules, contained in Judge Fletcher's Paper, "*On the different modes of reducing the apparent distance between the Moon and the Sun, or a Star, in Lunar Observations, to the true distance, for the purpose of ascertaining the Longitude.*" *By* Valentine Daintry, *Esq.*

Let A be the angle at the zenith; C, B the Moon and Star (or Sun); a, a′ the true and apparent distances between them; b, b′ the Moon's true and apparent altitudes; c, c′ those of the Sun or star:



RULE I.

We have $\sin.\frac{A}{2} = \sqrt{\frac{\sin.(S-b')\sin.(S-c')}{\sin. b' \sin. c'}}$: (S. T., Art. 24.)

Where $S = \frac{a' + b' + c'}{2}$.

$\cos. A = 1 - 2\sin^2.\frac{A}{2} = 1 - \frac{2\sin.(S-b')\ \sin.(S-c')}{\sin. b'\ \sin. c'}$

(P. T., Art. 43.)

$\cos. a, = \sin. b \sin. c \cos. A + \cos. b \cos. c.$

(S. T., Art. 22.)

$$= \sin. b \sin. c + \cos. b \cos. c - \frac{2 \sin. (S-b') \sin. (S-c')}{\sin. b' \sin. c'.} \times \sin. b \ \sin. c$$

$$= \cos. (b-c) - \frac{2 \sin. (S-b') \sin. (S-c')}{\sin. b' \sin. c'} \times \sin. b \sin. c.$$

This is the quantity calculated by Rule I., with the additional factor, $\frac{\sin\ c}{\sin. c'}$ in its second number, of which the Rule makes no mention. The log. of this (log. sin. c—log. sin. c′) is so nearly a constant quantity = .00012 or .00013, that the author has deemed fit to neglect its variation ; and augments the log. of 2, which enters the expression as a factor, by this small quantity. The parallax of the sum makes the slight difference between the constant numbers directed to be used, for the sun and star, respectively.

RULE II.

This is the same in principle as the last, the only difference being, that in place of the factor $\frac{\sin. b}{\sin b'.}$ its logarithm, ready calculated, in the Requisite Tables, is made use of. The quantity, .00012, added in the last Rule to the log. of 2, is here left out, as being included in the logarithms, in the tables referred to.

RULE III.

We have $1-2 \sin.^2 \tfrac{1}{2} a = \cos. a$ (P. T., Art. 43.)

$$\sin. \tfrac{1}{2} a = \sqrt{\frac{1-\cos. a}{2}}$$

$$=(\text{Rule I.})\sqrt{\frac{1-\cos. (b-c)}{2} - \frac{\sin. (S-b') \sin. (S-c')}{\sin. b \sin. c} \sin. b' \sin. c'}$$

$$= \sqrt{\underset{(\text{P.T., Art. 43.})}{\sin^2. \tfrac{1}{2} (b-c)} - \frac{\sin. (S-b') \sin. (S-c')}{\sin. b' \sin. c'} \sin. b \sin. c}$$

In order to reduce this expression to a logarithmic form, a subsidiary angle P is first calculated by the formula,

$$\frac{\sqrt{\frac{\sin. b}{\sin. b'}\sin. (S-b') \sin. (S-c')}}{\sin. \frac{1}{2} (b-c)} = \tan. P$$

To avoid the repetition of these lengthy expressions

$$\text{call this quantity} = \frac{m}{n} = \tan. P$$

$$\text{Then } \sin. P = \frac{\frac{m}{n}}{\sqrt{1 + \frac{m^2}{n^2}}} = \frac{m}{\sqrt{n^2 + m^2}}$$

Divide m by this quantity and we get

$$\sqrt{n^2 + m^2}$$

and substituting their values for m and n we get

$$\sqrt{\text{Sin.}^2 \tfrac{1}{2} (b-c) + \frac{\sin. b}{\sin. b'} \sin. (S-b') \sin. (S-c')}$$

the same expression as found above for sin. $\frac{1}{2}$ a.

It will be seen that the operations here gone through, are the same as those directed by Rule III., as, also, that the same allowance for $\frac{\sin. c}{\sin. c'}$ is made as in Rule I.

RULE IV.

$$\text{Cos. } (b'-c') - \cos\ a' = 2 \sin. \left(\frac{a' + b'-c'}{2}\right) \sin. \left(\frac{c' + a'-b'}{2}\right) \text{ (P. T., 41.)}$$

$$= 2 \sin. (S-c') \sin. (S-b')$$

The only difference between this Rule and the first, is, the substitution of the first for the last of these equivalents—.00012 representing as there the quantity $\frac{\sin. c}{\sin. c'}$

RULE V.

Is the same as last, only, as in Rule II., making use of log. $\left(\frac{\sin\ b}{\sin. b'}\right)$ ready calculated in the Requisite Tables.

RULE VI.

$$\text{Cos. A} = \frac{\text{cos. a}'-\text{cos. b}'\ \text{cos. c}'}{\text{sin. b}'\ \text{sin. c}'}$$

Find x from the equation,

$\text{cos. b}'\ \text{cos. c}' \quad \text{cos. } x' = \text{sin. } (90-x)$

$$\text{then cos. A} = \frac{\text{cos. a}'-\text{cos. } x}{\text{sin. b}'\ \text{sin. c}'} = \frac{2\ \text{sin. } \tfrac{1}{2}\ (x+\text{a}')\ \text{sin. } \tfrac{1}{2}\ (x-\text{a}')}{\text{sin. b}'\ \text{sin. c}'}$$

$$= \frac{2\ \text{cos } \tfrac{1}{2}\ ((90-\text{a}')+(90-x)) \times \text{sin. } \tfrac{1}{2}\ ((90-\text{a}')-(90-x))}{\text{sin. b}'\ \text{sin. c}'}$$

Now $\text{cos. a} = \text{cos A sin. b sin. c} + \text{cos. b. cos. c.}$ (S. T., 23.)

$$\text{cos. a} = \frac{2\ \text{cos. } \tfrac{1}{2}\ ((90-\text{a}')+(90-x)) \times \text{sin. } \tfrac{1}{2}\ ((90-\text{a}')+(90-x))\ \text{sin. b sin c}}{\text{sin. b}'\ \text{sin. c}'}$$

$+ \text{cos. b cos. c}$

Call the first term of this sin A

" Second........sin. B

then $\text{cos. a} = \text{sin. A} + \text{sin. B}$

$$= 2\ \text{sin. } \frac{\text{A}+\text{B}}{2}\ \text{cos. } \frac{\text{A}-\text{B}}{2} \quad \text{(P. T, 41)}$$

RULE VII.

Same as Rule VI., using log. $\frac{\text{sin. b}}{\text{sin. b}'}$ from Requisite Tables.

RULE VIII.

$$\text{Let} \sqrt{\frac{\text{sin. (S}-\text{b}')\ \text{sin. (S}-\text{c}')\ \text{sin. b sin. c}}{\text{sin. b}'\ \text{sin. c}'}} = \text{sin. P}$$

$$\text{By Rule III., sin. } \tfrac{1}{2}\ \text{a} = \sqrt{\frac{1-\text{cos. (b}-\text{c)}}{2} - \text{sin}^2\ \text{P}}$$

$$\text{cos. } \tfrac{1}{2}\text{a} = \sqrt{1-\text{sin.}^2\ \tfrac{1}{2}\ \text{a}} = \sqrt{\tfrac{1}{2} + \text{sin.}^2\ \text{P} + \frac{\text{cos. (b}-\text{c)}}{2}} =$$

$$\sqrt{\tfrac{1}{2}\ \text{cos. 2 P} + \tfrac{1}{2}\ \text{cos. (b}-\text{c)}} \quad \text{(P. T., 43.)}$$

Now, it may be easily ascertained, by comparison of the preceding operations, that the three precepts, which form the first part of this Rule, all lead to the same value, viz. :—

$$\text{Sin. of an arc } (P) = \sqrt{\frac{\text{sin. } (S - b') \text{ sin. } (S - c') \text{ sin. } b \text{ sin } c}{\text{sin. } b' \text{ sin. } c'}}$$

And by the second part of the rule,

$$\text{Cos. } \tfrac{1}{2}\, a = \sqrt{\text{cos. } \left(P - \frac{c}{2} + \frac{b}{2}\right) \times \text{cos. } \left(P + \frac{c}{2} - \frac{b}{2}\right)}$$

$$= \sqrt{\frac{\text{cos } 2P + \text{cos. } (b - c)}{2}} \text{ : The result found above. (P.T, 38.)}$$

N.B. References to the Articles of the "Elements of Plane and Spherical Trigonometry," in the "Library of Useful Knowledge." P.T., Plain Trigonometry. S.T., Spherical Trigonometry.

The "Literary and Historical Society" has lately been called upon, to deplore the early death of the amiable and talented Author of the above "Investigations." Though young in years, he possessed intellectual attainments of a very high order, and, was distinguished by his devotion to scientific pursuits. Notwithstanding the claims of a public office upon his time, he, yet, found leisure for his favorite studies, and, for zealous and efficient endeavours, for the advancement of the Society, of which he was long an active Member. But, this unremitting devotion to Science, gradually undermined a naturally weak constitution, and laid the foundation of the disease under which he sunk, on the 19th September, 1842, at Petworth, in Sussex, whither he had lately gone to try the effect of his native air upon his shattered health.